Math 105, Assignment 6 **Due 2015-04-08, 4:00 pm**

In the following problems you are expected to justify your answers unless stated otherwise. Answers without any explanation will be given a mark of zero. The assignment needs to be in my hand before I leave the lecture room or you will be given a zero on the assignment! **Don't forget to staple your assignment! You may lose a mark if you do not.**

1. Determine whether the following series converge or diverge:

   (a) $\displaystyle\sum_{s=2015}^{\infty} \frac{4^{2s}\pi^{-s}}{e^s}$

   (b) $\displaystyle\sum_{a=5}^{\infty} \frac{1}{a\log^{99} a}$

   (c) $\displaystyle\sum_{i=4}^{\infty} \frac{\sin(2015\log i)}{i^{2015}}$

   Hint: It may be a good idea to examine

   $$\sum_{i=4}^{\infty} \left|\frac{\sin(2015\log i)}{i^{2015}}\right|$$

   (d) $\displaystyle\sum_{f=1}^{\infty} \frac{(2f)!}{f^{2015}(f!)^2}$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{n+4n\sqrt{n+5\sqrt{n}}}{\sqrt[3]{8n^9-en^6+\pi n^3-2015}}$

   Hint: You may assume without proof that for large enough n

   $$a_n = \frac{n+4n\sqrt{n+5\sqrt{n}}}{\sqrt[3]{8n^9-en^6+\pi n^3-2015}}$$

   is positive and decreasing. Try a limit comparison test with

   $$b_n = \frac{1}{n^p}$$

   for some p, as done in class.

2. Suppose $\sum_{n=2}^{\infty} a_n$ is a geometric series. Suppose that $a_5 = 112, a_7 = 7$

   (a) Determine all possible values for the ratio, $r = \frac{a_{n+1}}{a_n}$

   (b) Suppose that $r < 0$, determine a_2, a_9.

   (c) Determine if the series converges and if it does, evaluate it.
   (**Note:** the series starts at $n = 2$ not $n = 0$)

3. Determine the center and radius of convergence of the following power series:

(a) $\sum_{n=3}^{\infty} \frac{n^2}{5^{n+1}}(x+10)^n$

(b) $\sum_{n=2}^{\infty} \frac{n!}{n^2 n^n}(y-1)^n$,
Hint: You may need to use the fact that

$$e = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n.$$

(c) $\sum_{n=1}^{\infty} \frac{2\cdot 4\cdot 6\cdots(2n-2)(2n)}{n!}x^n$

(d) **Bonus:**
$\sum_{n=0}^{\infty} f_n x^n$, where f_n is the Fibonacci sequence.

$$f_0 = 1 \quad f_1 = 1, \quad f_n = f_{n-1} + f_{n-2} \quad n \geq 2.$$

You may assume without proof that $\lim_{n\to\infty} \frac{f_{n+1}}{f_n}$ exists.
Fact: One can show that the above power series is the Taylor series of

$$\frac{1}{1-x-x^2}$$

4. Suppose we have

$$\sum_{n=1}^{\infty} \frac{5a_{n+1}n^3 - 4\sqrt{n}}{a_n n^3 + 6n^2 - 2} = \sqrt[e]{\pi^{2015}}$$

Determine the convergence of the series:

$$\sum_{n=1}^{\infty} a_n$$

Hint: Use the divergence test on the convergent series. What does that tell you about the limit

$$\lim_{n\to\infty} \frac{5a_{n+1}n^3 - 4\sqrt{n}}{a_n n^3 + 6n^2 - 2}$$

5. Find the power series and radius of convergence for the following functions:

(a) $\sin x$, centered at $x = \frac{\pi}{3}$

(b) $x^{2015}e^{x^2}$, centered at $x = 0$.

(c) $x^2 \log(2015 - x)$, centered at $x = 0$.

(d) $\frac{2015}{(2+x)^2}$, centered at $x = 1$.